CP VIOLATION IN FERMIONIC DECAYS OF HIGGS BOSONS*

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ABSTRACT

We demonstrate that decay angle correlations in $\tau^-\tau^+$ and $t\bar{t}$ decay modes could allow a determination of whether or not a neutral Higgs boson is a CP eigenstate. Sensitivity of the correlations is illustrated in the case of the $e^+e^- \to ZH$ and $\mu^+\mu^- \to H$ production processes for a two-doublet Higgs model with CP-violating neutral sector. A very useful technique for minimizing 'depolarization' factor suppressions of the correlations in the $t\bar{t}$ mode is introduced.

Determination of the CP nature of any neutral Higgs boson that is directly observed will be crucial to fully unraveling the nature of the Higgs sector. It is entirely possible to have either explicit or spontaneous CP violation in the neutral Higgs sector. Indeed, the simplest non-supersymmetric two-Higgs-doublet model (2HDM) and the supersymmetric Higgs two-doublet plus singlet model both allow for Higgs mass eigenstates of impure CP nature[1]. Here we shall focus on the 2HDM, in which CP violation results in three neutral states, $H_{i=1,2,3}$, of mixed CP character.

The most direct probe of CP violation is provided by comparing the Higgs boson production rate in collisions of two back-scattered-laser-beam photons of various different polarizations[2]. A certain difference in rates for different photon helicity choices is non-zero only if CP violation is present, and has a good chance of being of measurable size for many 2HDM parameter choices.

Correlations between decay products can also probe the CP nature of a Higgs boson. In this paper we focus on effects that arise entirely at tree-level. For the dominant two-body decays of a Higgs boson, one can define appropriate observables if we are able to determine the rest frame of the Higgs boson and if the secondary decays of the primary final state particles allow an analysis of their spin or helicity directions. An obvious example is to employ correlations between the decay planes of the decay products of WW or ZZ vector boson pairs and/or energy correlations among the decay products[4,5,6,7,8,9,10]. However, these will not be useful for a purely CP-odd H (which has zero tree-level WW, ZZ coupling and thus decays primarily to $F\overline{F}$) or for

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a mixed-CP H in the (most probable) case where the CP-even component accounts for essentially all of the WW, ZZ coupling strength (thereby yielding 'apparently CP-even' correlations). In contrast, H decays to $\tau^-\tau^+$ or $t\bar{t}$, followed by τ or t decays, do, in principle, allow equal sensitivity to the CP-even and CP-odd components of a given Higgs boson.

Indeed, a H eigenstate couples to $F\overline{F}$ according to: $\mathcal{L} \propto \overline{F}(a+ib\gamma_5)FH$,which yields

$$\langle F_+\overline{F}_+|H\rangle \propto b + ia\beta_F; \quad \langle F_-\overline{F}_-|H\rangle \propto b - ia\beta_F\,, \tag{1}$$

where $\beta_F = \sqrt{1-4m_F^2/m_H^2}$, helicity-flip amplitudes being zero. The crucial point is that, in general, a and b are of comparable magnitude in a CP-violating 2HDM.

Here, we shall present a unified treatment aimed at realistically evaluating the possibility of using correlations in $H \to \tau^-\tau^+$ and $t\bar{t}$ final states to determine if a decaying Higgs boson is a mixed CP eigenstate, thereby directly probing for the presence of CP violation in the Higgs sector.

An efficient framework for our analysis is that developed in Refs. [11,3]. Consider the charged current decay $F \to Rf$, where F is a heavy fermion, f is a light fermion whose mass can be neglected, and R can be either a single particle or a multiparticle state with known quantum numbers and, therefore, calculable coupling to the charged weak current. (Examples are $\tau \to R\nu$, where $R = \pi, \rho, A_1, \ldots$, and $t \to Wb$, where W decays to a fermion plus anti-fermion.) For the R's of interest, the form of the hadronic current J_μ, deriving from the standard $V-A$ interaction for the $J_\mu \equiv \langle R|V_\mu - A_\mu|0\rangle$ coupling, is completely determined in terms of the final particle momenta. Using the particle symbol to denote also its momentum, and defining

$$\Pi_\mu = 4\mathrm{Re}\, J_\mu f\cdot J^* - 2f_\mu J\cdot J^*, \quad \Pi^5_\mu = 2\epsilon_{\mu\rho\nu\sigma}\mathrm{Im}\, J^\rho J^{*\,\nu} f^\sigma, \tag{2}$$

all useful correlations in $H \to F\overline{F}$ decay can be obtained by employing the quantities

$$\omega = F\cdot(\Pi - \Pi^5), \quad R_\mu = m_F^2(\Pi-\Pi^5)_\mu - F_\mu F\cdot(\Pi-\Pi^5)\,, \tag{3}$$

and their $\overline{F}$ analogues. In the F rest frame, $R_0 = 0$, $\vec{R} = m_F^2(\vec{\Pi}-\vec{\Pi}^5)$, and $|\vec{R}| = m_F\omega$. In fact, $\vec{S}_F = \vec{R}/(m_F\omega)$ acts as an effective spin direction ($|\vec{S}_F|^2 = 1$) when in the F rest frame.

Let us give some illustrative examples. For $\tau^- \to \pi^-\nu$ decay, $J_\mu \propto \pi^-_\mu$ and $\vec{S}_{\tau^-} = \hat{\pi^-}$ is the unit vector pointing in the direction of the π^-'s three momentum (using angles defined in the τ^- rest frame). For $\tau^- \to \rho^-\nu \to \pi^-\pi^0\nu$, $J_\mu \propto (\pi^- - \pi^0)_\mu$, yielding $\Pi_\mu \propto 4(\pi^- - \pi^0)_\mu \nu\cdot(\pi^- - \pi^0) + 2\nu_\mu m_\rho^2$, and, thence, $\vec{S}_F \propto m_\tau(\vec{\pi^-} - \vec{\pi^0})(E_{\pi^-} - E_{\pi^0}) + \vec{\nu}m_\rho^2/2$, where the pion energies and directions are defined in the τ^- rest frame. For $t \to W^+b \to l^+\nu b$, $J_\mu \propto \overline{u}(\nu)\gamma_\mu(1-\gamma_5)v(l^+)$, and $\Pi_\mu \propto l^+_\mu \nu\cdot b + \nu_\mu l^+\cdot b$, $\Pi^5_\mu \propto \nu_\mu l^+\cdot b - l^+_\mu\nu\cdot b$, so that $\Pi_\mu - \Pi^5_\mu \propto l^+_\mu$, implying $\vec{S}_t = \hat{l}^+$ in the t rest frame.

If the full $(\Pi - \Pi^5)_\mu$ can be determined on an event-by-event basis, then we can define the 'effective spin' vectors $\vec{S}_F$ and $\vec{S}_{\overline{F}}$ *for each event*, and the distribution of

the Higgs decay products takes the very general form

$$dN \propto \Big[(b^2+a^2\beta_F^2)(1+\cos\theta\cos\overline{\theta})+(b^2-a^2\beta_F^2)\sin\theta\sin\overline{\theta}\cos(\phi-\overline{\phi}) \\ -2ab\beta_F\sin\theta\sin\overline{\theta}\sin(\phi-\overline{\phi})\Big]d\cos\theta d\cos\overline{\theta}d\phi d\overline{\phi}\,, \quad (4)$$

where θ,ϕ and $\overline{\theta},\overline{\phi}$ define the angles of $\vec{S}_F$ and $\vec{S}_{\overline{F}}$ in the F and $\overline{F}$ rest frames, respectively, *employing the direction of F in the H rest frame as the coordinate-system-defining z axis.*

If we cannot determine $(\Pi-\Pi^5)_\mu$ for each event, then Eq. 4 must be modified. An extreme example is $F\to Rf$ decay where the R decay products are not examined. In this case the angles of R in the F rest frame would be employed in Eq. 4, and 'depolarization' factors arise as a result of event averaging. In deriving Eq. 4, the angular independent term is actually multiplied by $(m_F\omega_F)(m_F\omega_{\overline{F}})$ and the $\cos\theta\cos\overline{\theta}$, $\sin\theta\sin\overline{\theta}\sin(\phi-\overline{\phi})$ and $\sin\theta\sin\overline{\theta}\cos(\phi-\overline{\phi})$ terms by $|\vec{R}_F||\vec{R}_{\overline{F}}|$. On an event-by-event basis the ratio of these coefficients is unity, as outlined earlier. When averaged over events, this is no longer true. Consequently, when event averaging (denoted by $\langle\ldots\rangle$) all the angle-dependent terms in Eq. 4 must be multiplied by $D_F\equiv\langle|\vec{R}_F|\rangle/(m_F\langle\omega_F\rangle)$ and/or its $D_{\overline{F}}$ analogue, relative to the angle-independent term. We define $D\equiv D_FD_{\overline{F}}$.

At first sight, the necessity of event averaging arises in the case of the $t\overline{t}$ final state, for which we will find that we must have one top decay leptonically and the other hadronically in order to define the $t\overline{t}$ line of flight and, thereby, appropriate angles in Eq. 4. For the hadronically decaying top, the problem is to distinguish the quark vs. anti-quark jet coming from the W so as to construct $(\Pi-\Pi^5)_\mu$ (which is proportional to the W^+ (W^-) anti-quark (quark) momentum for t ($\overline{t}$) decay) for each event. If we simply sum over all W decay product configurations, then the appropriate depolarization factor is easily computed by using $J_\mu\propto\epsilon_\mu^W$ and summing over W polarizations. One finds $D_t=(m_t^2-2m_W^2)/(m_t^2+2m_W^2)\sim 0.4$ for $m_t=174$ GeV. Similarly, for $\tau\to R\nu$ where R is spin-1.

Let us now specify our procedure for isolating the coefficients of the $\cos(\phi-\overline{\phi})$ and $\sin(\phi-\overline{\phi})$ angular correlation terms. Defining $c\equiv\cos\theta$, $\overline{c}\equiv\cos\overline{\theta}$, $s\equiv\sin\theta$, $\overline{s}\equiv\sin\overline{\theta}$, $c_\phi\equiv\cos\delta\phi$, $s_\phi\equiv\sin\delta\phi$ (where $\delta\phi\equiv\overline{\phi}-\phi$), and $d\Omega\equiv dc\,d\overline{c}\,d\delta\phi$, and including a possible depolarization factor, we have

$$\frac{1}{N}\frac{dN}{d\Omega}=\frac{1}{8\pi}\left[1+Dc\overline{c}+\rho_1 s\overline{s}s_\phi+\rho_2 s\overline{s}c_\phi\right]\,, \quad (5)$$

where

$$\rho_1\equiv D\frac{2ab\beta_F}{(b^2+a^2\beta_F^2)}\,,\quad \rho_2\equiv D\frac{(b^2-a^2\beta_F^2)}{(b^2+a^2\beta_F^2)}\,. \quad (6)$$

For a CP-conserving Higgs sector, either $a=0$ or $b=0$ implying $\rho_1=0$ and $|\rho_2|=D$. For a CP-mixed eigenstate, both a and b are non-zero. Thus $\rho_1\neq 0$

provides an unequivocable signature for CP violation in the Higgs sector, while the difference $D - |\rho_2|$ also provides a measure of Higgs sector CP violation. (Indeed, ρ_1 and ρ_2 are not independent; $\rho_1^2 + \rho_2^2 = D^2$.) Values of $\rho_1 \sim D$ and $\rho_2 \sim 0$ are common in an unconstrained 2HDM.

To isolate ρ_1 and ρ_2, we define projection functions $f_{1,2}(\theta, \overline{\theta}, \delta\phi)$ such that $\int f_{1,2} d\Omega = 0$, $\int f_{1,2} c\overline{c} d\Omega = 0$, $\int f_1 s\overline{s} s_\phi d\Omega = 8\pi$, $\int f_1 s\overline{s} c_\phi d\Omega = 0$, $\int f_2 s\overline{s} s_\phi d\Omega = 0$, and $\int f_2 s\overline{s} c_\phi d\Omega = 8\pi$. Then, $\rho_{1,2} = \int f_{1,2} \frac{1}{N}\frac{dN}{d\Omega} d\Omega$. The critical question is with what accuracy can $\rho_{1,2}$ be determined experimentally? The error is minimized by using projection functions which match the angular dependence of the term of interest. Thus, we employ $f_1 = (9/2) s\overline{s} s_\phi$ and $f_2 = (9/2) s\overline{s} c_\phi$, for which $y_{1,2} = 9/2$ and $\rho_{1,2}/\delta\rho_{1,2} = \sqrt{2/9}\rho_{1,2}\sqrt{N}/[1 - (2/9)\rho_{1,2}^2]^{1/2}$.

We now discuss the Higgs production reactions and the $\tau^-\tau^+$ and $t\overline{t}$ final state decay modes for which the angles of Eq. 4 can be experimentally determined. Consider first the $\tau^-\tau^+$ case. The τ decays are of two basic types: $\tau \to l\nu\nu$ and $\tau \to R\nu$, where R is a hadronically decaying resonance of known quantum numbers. Together these constitute about 95% of the τ decays, with $BR(\tau \to \Sigma R\nu) \sim 58.8\%$. Thus, we employ $D = 1$ and an effective branching ratio for useful $\tau^-\tau^+$ final states of $(0.588)^2$.

In the case of $t\overline{t}$ decays, we employ only the case where one top decays hadronically, and the other leptonically, such that we are simultaneously able to determine the exact $t\overline{t}$ decay axis and distinguish t from $\overline{t}$. Thus, we adopt an effective branching ratio for useful $t\overline{t}$ final states of $2 \times (2/3) \times (2/9)$ (keeping only $l = e, \mu$). Employing *one* hadronic t (or $\overline{t}$) decay and identifying the most energetic jet from the W^+ (W^-) with the anti-quark (quark) leads to a depolarization factor of $D \sim 0.78$.

In order to assess our ability to experimentally measure ρ_1 and ρ_2, we have examined H production in the reactions $e^+e^- \to ZH$ at a future linear e^+e^- collider and $\mu^+\mu^- \to H$ at a possible future $\mu^+\mu^-$ collider[12].

For the e^+e^- collider we have adopted the optimal energy, $\sqrt{s} = m_Z + \sqrt{2} m_H$, and assumed an integrated luminosity of 85 fb^{-1}. For the $\mu^+\mu^-$ collider we have computed the Higgs signal and the continuum $\tau^-\tau^+$ and $t\overline{t}$ backgrounds assuming unpolarized beams and a machine energy resolution of 0.1%, with $\sqrt{s}$ centered at the value of m_H.

For m_H values such that the $e^+e^- \to ZH$ production mode is background free, the statistical significance of a non-zero result for ρ_1 is that given earlier, $N_{SD}^1 = |\rho_1|/\delta\rho_1$, where $\delta\rho_1 = (9/2 - \rho_1^2)^{1/2}/\sqrt{N}$, and N is the number of events *after including the branching ratios required to achieve the final state of interest*: $BR_{eff} = BR(H \to F\overline{F}) \times BR(F\overline{F} \to X)$, where the latter $F\overline{F}$ branching ratios to useful final X states were specified above. In the case of ρ_2 we must actually determine the statistical significance associated with a measurement of $D - |\rho_2|$. This is given by $N_{SD}^2 = [D - |\rho_2|]/\delta\rho_2$, where $\delta\rho_2 = (9/2 - \rho_2^2)^{1/2}/\sqrt{N}$.

In $\mu^+\mu^- \to H$, the continuum backgrounds must be included. The statistical significance of a non-zero value for ρ_1 is given by $N_{SD}^1 = |\rho_1|/\delta\rho_1$ with $\delta\rho_1 = [9/2 -$

Figure 1: The maximum statistical significances N_{SD}^1 and N_{SD}^2 for $H \to \tau^-\tau^+$ (———) and $H \to t\bar{t}$ (– – – –), in $e^+e^- \to ZH$ ($L = 85$ fb^{-1}) and $\mu^+\mu^- \to H$ ($L = 20$ fb^{-1}) production, after searching over all α_1 and α_3 values at fixed m_H and $\tan\beta$. In each case, curves for the three $\tan\beta$ values of 0.5, 2, and 20 are shown. In the $\tau^-\tau^+$ ($t\bar{t}$) mode N_{SD} values increase (decrease) with increasing $\tan\beta$, except in the case of $\mu^+\mu^- \to H \to t\bar{t}$, where the lowest curve is for $\tan\beta = 0.5$, the highest curve is for $\tan\beta = 2$, and the middle curve is for $\tan\beta = 20$.

$\rho_1^2 + (B/S)(9/2+\rho_1^2)]^{1/2}/\sqrt{S}$, where S is the total number of events from H production, and B is the total number of events from the continuum background, in the final state of interest. For ρ_2 we have $N_{SD}^2 = [D - |\rho_2|]/\delta\rho_2$ with $\delta\rho_2 = [9/2 - \rho_2^2 + (B/S)(9/2 + \rho_2^2 - 2\rho_2\rho_2^B)]^{1/2}/\sqrt{S}$, where ρ_2^B is that for the background alone.

Our results for the maximum N_{SD}^1 and N_{SD}^2 values are presented in Fig.1, where we have adopted a top quark mass of 174 GeV. The maximum values were found by searching (holding $\tan\beta$ and m_H fixed) over all values of the Higgs sector mixing angles α_1 and α_3, (for the notation see Ref. [13]).

Consider first the results for $e^+e^- \to ZH$ collisions. From Fig. 1 we find that detection of CP violation through both ρ_1 and ρ_2 is very likely to be possible for $m_H < 2m_W$ via the $H \to \tau^-\tau^+$ decay mode. This is an important result given that various theoretical prejudices suggest that the lightest Higgs boson is quite likely to be found in this mass range. For m_H between $2m_W$ and $2m_t$, a statistically significant measurement of CP violation will be difficult. For $m_H > 2m_t$, detecting CP violation in the $t\bar{t}$ mode would require a somewhat larger L (of order 5 times the assumed luminosity of $L = 85$ fb^{-1} for $\tan\beta$ between 2 and 5).

In $\mu^+\mu^- \to H$ production, Fig. 1 shows that the maximum N_{SD}^1 and N_{SD}^2 values in the $\tau^-\tau^+$ mode can remain large out to large Higgs masses if $\tan\beta$ is large, but that for small to moderate $\tan\beta$ values the statistical significances are better in $e^+e^- \to ZH$ collisions when $m_H < 2m_W$. However, Fig. 1 also indicates that the $\mu^+\mu^- \to H \to \tau^-\tau^+$ channel has the advantage of possibly small sensitivity to the WW decay threshold at $m_H \sim 2m_W$. Such insensitivity arises when the Euler angles α_1, α_3 are chosen so as to minimize WW, ZZ couplings (and hence $H \to WW, ZZ$ branching ratios) without sacrificing production rate. Thus, for $L = 20$ fb^{-1} $\mu^+\mu^-$ collisions could allow detection of CP violation all the way out to $2m_t$ for $\tan\beta \gtrsim 10$. The $t\bar{t}$ final state extends the range of m_H for which detection of CP violation might be possible only somewhat, and only if $\tan\beta$ lies in the moderate range near 2.

Acknowledgments

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